Modeling of Rarefied Gas Flows on the Base of Numerical Solving of the Boltzmann Equation

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The traditional approach of gas flow modeling is based on Euler or Navier-Stocks equations. This approach is relevant if the mean free path of molecules is small compared to specific size of flows, whereas the rarefied gas effects are not important. However, due to development of micro devices in various application areas, at the present time interest in rarefied gas modeling has been increasing. A kinetic approach is required to describe the rarefied gas flows. The basis of the kinetic theory of gases is the Boltzmann kinetic equation.

The complexity and non-linearity of the Boltzmann equation cause many researchers to use alternative approaches. One of these is the Direct Simulation Monte-Carlo (DSMC) method. However, because of high level of statistical noise the Monte-Carlo methods are not very efficient for simulating slightly disturbed flows, which are typical for microdevices. Other prevalent approaches are based on using model kinetic equations where the right term of the Boltzmann equation – the collision integral – is replaced by simpler relaxation forms with less computational complexity. Nevertheless, the reliability of results obtained by these methods is unknown.

Nowadays, the growth of the power of modern computation systems made it possible to solve the Boltzmann kinetic equation directly without any simplifications. In the present work we solve the Boltzmann equation by finite-difference method with application of the conservative projection method for calculation of the collision integral [1]. The last method ensures that the laws of conservation of mass, momentum and energy are strictly observed, and the collision integral of the Maxwellian distribution function is equal to zero. Real molecular potentials can be used in simulation. The method can be applied to a single component gas, a gas mixture [2] and a gas with internal degrees of freedom [3].

On the basis of the conservative projection method the problem-solving environment (PSE) designed for simulation of rarefied gas flows was developed. The PSE allows one to perform calculations on personal computer, including the use of graphics processors [4]. Complex two- and three-dimensional problems are computed on multi-processor clusters.

In course of development of the program code some bottlenecks were located using performance analyzers. The code was optimized by using of SSE and AVX instructions that gives about 40-50% increase of the speed of computations. A version adapted to Intel MIC architecture is now under development.

The work presents the results of simulation of some slightly disturbed flows as well as supersonic flows. Examples of slightly disturbed flows show modeling of Knudsen pump and its multistage modification. The accuracy of obtained results is proved by parametric computations and is limited only by the available computational resources.

References

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